Università degli Studi dell'Insubria

Corso di Laurea in Matematica- Corso di Laurea Specialistica in Matematrica
Analisi Numerica III - Metodi Numerici in Informatica

Prima parte: Modulo "Numerical linear algebra in image restoration"
Docente: Claudio Estatico

Registro delle lezioni

<u>DATE</u>   <u>h.</u>

March (1 week)

Wednesday	23	9-11	2	Introduction to Inverse Problems in Imaging. Image Deblurring and basic regularization in discrete setting via Truncated SVD. [III, Ch.1 Sec.1.1 p.1-3, Sec.1.4 p.8-12]
Thursday	24	9-11	2	The Fredholm equation of the first kind as basic model in continuous setting. Hadamard's well-posedness. Ill-posed problems. [I, Ch.2 Sec.2.1 Sec.2.2 p.21-24] OR [VI, Ch.1 Sec.1.1 p.1-4] OR [II, Ch.4 Sec.4.1 p.75-78] Generalized Inverse [I, Ch.1 Sec.1.1 Sec.1.2 p.5-7] OR [VI, Ch.1 Sec.1.2 p. 5-13]

April (4 weeks)

Wednesday	6	9-11	2	Generalized inverse, variational vs. operational approach. Properties. [I, Ch.1 Sec.1.3 Sec.1.4 p.8-17]
Thursday	7	9-11	2	Compact operators. Generalized inverse of compact operators via singular system. [I, Ch.1 Sec.1.5 p.17-18; I App.A p. 117-118], [V, Ch. 4 p.136-148]
Friday	8	9-13	4	Picard's conditions. Generalized inverse of matrices. [I, Ch.1 Sec.1.5 p.17-20]. Regularization algorithms. Convergence of the regularized solution for true data. Divergence of the regularized solution for true data. Tikhonov regularization, Tikhonov functional and Euler equation. [I, Ch.2 Sec.2.3 Sec.2.4 p.24-28] OR [VI, Ch.2 Sec.1.2 p. 15-17]. Residual norm, regularized solution norm vs. regularization parameter. [I, Ch.2 Sec.2.5 p.30-36], [VI, Ch.2 Sec.1.2 p. 15-17]
Wednesday	13	9-11	2	Bounded residual, bounded solutions. Approximation error and noise data restoration error. Optimal regularization parameter. [I, Ch.2 Sec.2.5 p.30-36] OR [II, Ch.5 Sec.5.1-5.4 p.98-112].
Thursday	14	9-11	2	Discrepancy principle . [I, Ch.2 Sec.2.5 p.37] OR [VI, Ch.1 Sec.3.3 p.43-45]. Spectral filtering. [I, Ch.2 Sec.2.6 p.39,42-44]. Iterative regularization algorithms. Landweber method. [I, Ch.2 Sec.2.7 p. 45-50].
Tuesday	19	9-11	2	Steepest descent method. [I, Ch.3 Sec.3.2 p. 64-68] OR [II, Ch.6 Sec.6.4 p.157-165].

Wednesday	20	9-11	2	Conjugate Gradient Method. [I, Ch.3 Sec.3.3 p. 69-84] OR [II, Ch.6 Sec.6.4 p.157-165]. Spectral properties.
Thursday	28	9-11	2	Preconditioning. Discretization of ill-posed problems [I, Ch.3 Sec.2.8 p. 50-51]. Signal and image deblurring.
Friday	29	9-13	4	Boundary conditions. Fast trigonometric transformations. Circulant matrices, diagonalization. Toeplitz convolution via circulant convolution by FFT. 1d and 2d cases.

May (1 week)

Thursday	5	9-11	2	Incomplete Choleski factorization. Optimal and Superoptimal Preconditioning. Convergence speed of CG and spectral properties. Cluster and outliers.
Wednesday	11	9-11	2	CG, Natural Preconditoning and Banded Toeplitz. Talk dr. Frandi (Regularization in Banach spaces and image restoration).
		Total h.:	30	

Bibliography

I. C. Estatico,
    Gradiente coniugato e regolarizzazione di problemi mal posti
    Quaderni del Gruppo Nazionale per l'Informatica Matematica, n.6, Firenze, 1996.
    Il testo non è più reperibile. Ho messo una copia elettronica (3 file PDF) nella pagina del corso
    Testo teorico.

II. M. Bertero, and P. Boccacci,
     Introduction to Inverse Problems in Imaging
     IOP Publishing, Bristol, 1998.
     Testo teorico, con richiami all'implementazione (ossia alla pratica). È un testo base sull'image restoration per l'intera comunità scientifica.

III. P. C. Hansen, J. Nagy, and D. P. O'Leary
      Deblurring Images: Matrices, Spectra and Filtering
      SIAM, Philadelphia, Pa, USA, 2006.
      Testo dedicato alla pratica.

IV. C.W. Groetsch,
     Generalized Inverses of Linear Operators: Representation and Approximation
     Dekker, New York, 1977.
     Testo teorico.

V. C.W.Groetsch,
    Elements of Applicable Functional Analysis
    Dekker, New York, 1980.
    Testo teorico.

VI. C.W. Groetsch,
     The Theory of Tikhonov Regularization for Fredholm Equations of the First Kind,
     Pitman, London, 1984.
     Testo teorico.

VII. C.W. Groetsch,
      Stable Approximate Evaluation of Unbounded Operators,
      Springer Verlag, Heidelberg and Berlin, Germany, 2007.
      Testo teorico.