

Numerical linear algebra: tools and methods

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1. Some tools

- (a) The singular value decomposition
- (b) The bordering method
- (c) The Schur complement
- (d) The Sherman-Morisson formula
- (e) Sylvester identity

2. Topics on linear systems

- (a) Preconditioning
- (b) Regularization

3. Projection methods for linear systems

- (a) Steepest descent
- (b) The conjugate gradient
- (c) Krylov subspace methods

4. An application: web search

- (a) The Google matrix
- (b) The PageRank vector
- (c) The power method
- (d) Convergence acceleration
- (e) Extrapolation

Numerical linear algebra is a very important chapter of numerical analysis since, in many situations and applications, one has to solve a system of linear equations or to compute the eigenvalues and the eigenvectors of a matrix.

In these lectures, we will first present some useful tools in this domain that could be used either to prove theoretical results or for computational purposes.

Then, we will introduce the notions of preconditioning and regularization which are related to the finite precision of the computations on a computer.

Nowadays, very large systems of linear equations (over 10^6) have to be solved. For treating them, one has to use iterative methods such as projections methods. After presenting the methods of steepest descent and conjugate gradient, we will concentrate on Krylov subspace methods which are now commonly used. Some of these methods are quite new.

When key words are introduced into a web search engine, the answers are given in a decreasing order of relevance. A mathematical problem is behind this search. It will be described, and the numerical methods for its solution will be presented.