In this book we study the direct and inverse scattering theory of the Zakharov-Shabat system. The direct problem consists of deriving the scattering data (the reflection coefficient, the bound states and the norming constants) starting from the potentials \( k(x) \) and \( l(x) \). The analytic and continuity properties of the Jost solutions and the scattering data are established in a rigorous way. The inverse scattering theory of determining the potentials when the scattering data are given is formulated in terms of the Marchenko equations. The main problem to get an explicit solution of the Marchenko equations is solved by using matrix triplets. Finally, the Inverse Scattering Transform is applied to solve the initial value problem for the Nonlinear Schrödinger equation (NLS). We derive an explicit solution formula of the NLS equation which includes many of the \( N \)-soliton solutions already known in the literature and a new class of solutions: the multipole solutions (corresponding to bound states with algebraic multiplicities greater than one).